Mathematics For Economists
Module 1, academic year 2019–2020

Sergei Golovan
New Economic School
sgolovan@nes.ru

TA: Igor Samsonov (isamsonov@nes.ru)

Course description

The course “Mathematics for Economists” is designed to introduce the students to mathematical and programming tools which are widely used in economics, particularly in micro- and macroeconomics lecture courses. The course is a compulsory one, and is taught at the first module of the first year. It consists of 14 lectures and 7 seminars.

Course requirements, grading, and attendance policies

The course doesn’t have any special prerequisites except for the standard calculus and linear algebra courses.

There will be 5 home assignments which will constitute 20% of the final grade. The final exam will account for the remaining 80%.

Course contents

1. Preliminaries
   (a) The intermediate and mean value theorems
   (b) The inverse and implicit function theorems
   (c) The basics of discrete dynamic systems
   (d) The basics of ordinary differential equations
2. Finite-dimensional optimization
   (a) Unconstrained optimization problem
   (b) Equality-constrained optimization problem, theorem of Lagrange
   (c) Unequality-constrained optimization problem, theorem of Kuhn and Tucker
   (d) Convexity and optimization
3. Parametric optimization and comparative statics

   (a) Differential comparative statics
   (b) Monotone comparative statics
   (c) Continuous comparative statics

4. Dynamic programming, Bellman's equations
   (a) Finite horizon dynamic programming
   (b) Stationary discounted dynamic programming

5. Optimal control in continuous time, Pontryagin's maximum principle

6. Fixed point theorems and existence of equilibria
   (a) Fixed points of contraction mappings
   (b) Fixed points of continuous mappings

Description of course methodology

Lectures will proceed from motivating examples and sample models in economics to general principles of mathematical modeling.

Sample tasks for course evaluation

1. Find the set of all solutions in the following optimization problem as a function of $w > 0$:

$$\max x^{1/4}y^{1/4} + z \quad \text{s.t. } w - x - y - z \geq 0,\ x, y, z \geq 0.$$

2. Solve the following optimization problem as a function of $\alpha > 0$:

$$\max \alpha x + y \quad \text{s.t. } y + (x - 1^3) \leq 0,\ x, y \geq 0.$$

3. Consider the following optimal control problem:

$$J = \int_0^1 \left(1 - e^{-c(t)-s(t)}\right) dt \to \max$$
$$\begin{aligned} \text{s.t. } \dot{s}(t) &= -c(t), \\ c(t) &\geq 0,\ t \in [0,1], \\ s(1) &\geq 0, \\ s(0) &= s_0 > 0 \text{ is given.} \end{aligned}$$

   In this problem savings $s(t)$ is a state variable, consumption $c(t)$ is a control variable, time $t \in [0,1]$.

   (a) Formulate the Hamiltonian for this problem. Derive the first order condition and transversality condition. Could it be that the costate variable is non-positive for this problem?
   (b) Plot the phase diagram (using state and costate variables as coordinates) for this problem and put the optimal trajectory on it. Where are the starting and ending points of the optimal trajectory?

(c) Under which values of $s_0 > 0$ the control variable constraint $c(t) \geq 0$ is not limiting for all $t \in [0,1]$? Derive the optimal control trajectory $c(t)$, $t \in [0,1]$, for this case.

(d) Under which values of $s_0 > 0$ the optimal control switches between regimes $c(t) = 0$ and $c(t) > 0$ when $t \in (0,1)$? Derive the equation which specifies the switching moment t^*. Show that t^* is defined unambiguously. Derive the optimal control trajectory $c(t)$, $t \in [0,1]$, for this case.

(e) Consider now another problem in which there's no constraint $s(1) \geq 0$ (so, the agent is allowed to be indebted at the end of her life), but instead of J she maximizes $J + e^{-1}s(1)$. Derive the optimal control trajectory $c(t)$, $t \in [0,1]$, for this case. Under which values of s_0 the agent doesn't consume at all?

Course materials

1. Acemoglu D. (2009) Introduction to Modern Economic Growth, Princeton University Press.
2. Ljungqvist L., T. J. Sargent (2012) Recursive Macroeconomic Theory, Third Edition, MIT Press.
3. Rudin, W. (1976) Principles of Mathematical Analysis, Third Edition, McGraw-Hill International Editions, Singapore.
4. Sundaram R. K. (1996), A First Course in Optimization Theory, Cambridge University Press.

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.